Network Application of Cooperative Game Theory

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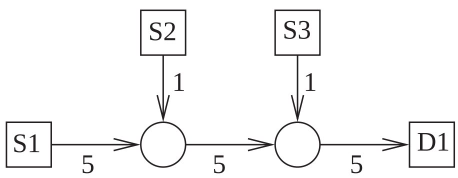
September 25, 2003

The emergence of packet switched networks has raised a number of new challenges to the scientific community. In particular, the question of how should the network resources be shared between users is a very important one. Let's consider the simple introductory example shown in Fig. 1 showing three sources and one destination. Each link has a finite capacity and the table on the right shows different possible sending rates for the sources :



	S1	S2	S3	tot. utilisation
1.	3	1	1	14
2.	4	1	0	15
3.	4	0	1	14
4.	5	0	0	15

Figure 1: Example of rate allocation

All the rate allocations shown in the table above correspond to situations where, given the maximum capacity of each link, no user can increase its emission rate without decreasing the rate of another user. Such a solution is called "Pareto optimum" and is clearly desirable in computer networks. In general, there exists an infinity of Pareto optimum solutions for a given problem. Therefore, additional constraints must be introduced. By inspection of the table above, it can be seen that the rate allocation corresponding to line 2. and 4. yield a total link utilisation which is maximum. However, these allocations prevent S3 from accessing the network. Clearly, the first line of the table gives the most attractive solution because it gives a fair share of the network resources to each user. Fairness between user is very important and has to be considered when selecting Pareto-optimum solution. In practice, the problem of rate allocation is worsened by the fact that the different sending rate are not distributed by the network but are instead selected by each user who may react to other's decisions.

In this presentation, we illustrate how this problem can be presented in a game theoretic framework by way of examples drawn from the literature : in [3] and [4] the *Nash Bargaining Solution*(NBS) is investigated. This solution is not only Pareto optimal but also satisfies certain axioms of fairness. We note that this solution is different from Nash equilibrium which, in general, are not Pareto optimum [1]. The NBS corresponds to what has been called *proportional fairness*. Therefore, different notions of fairness are briefly discussed following the text from F. Kelly on the fairness of End-to-End congestion control [2].

References

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